

# Complete Spatial Randomness and Quadrat Methods

GRASS Tutorial on `s.qcount`

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08 January 1993

Cressie [1] defines the concept of *complete spatial randomness* (csr) as synonymous with a *homogeneous* Poisson process in  $\mathbb{R}^d$  (here the concern is  $d = 2$ ). In layman's terms, the definition states that events are equally likely to occur anywhere within an area  $A \subset \mathbb{R}^d$ .

There are two types departure from a csr: regularity and clustering. Figure 1 shows realizations of three processes for  $N(A) = 50$ . Notice in figure 1a that this process may *appear* to be clustered. This is because event-to-nearest-event distances ( $W$ ) of a homogeneous Poisson process can be modeled as  $\chi_2^2$  random variables. Recall the form of the  $\chi_2^2$  probability density function (fig. 2). Most of the probability is near zero. It follows that, given a suitable statistic, csr can be tested using Pearson's  $\chi^2$  goodness-of-fit test.

Various indices and statistics measure departure from csr, ie., the pattern. Tables 8.3 and 8.6 in Cressie's book [1] summarizes six *quadrat count* indices and 17 *nearest neighbor* test statistics for quantifying departure from csr. The former, also discussed in Ripley [7], are implemented in `s.qcount` and described later.

`s.qcount` chooses  $n$  circular regions of radius  $r$  such that they are completely within the bounds of the current geographic region and no two regions overlap. The regions are called random *quadrats*.<sup>1</sup> The number of sites falling within each quadrat are counted and indices are calculated to estimate the departure of site locations from complete spatial randomness. This is illustrated in figure 3.

The six indices and their realizations for the sampling shown in figure 3 are shown in Table 1. Cressie [1] gives a short summary of these on pages 590 and 591 and Ripley [7] discusses them on pages 102–106. The original reference to each index is given in Table 1 in case you want to read more about them.

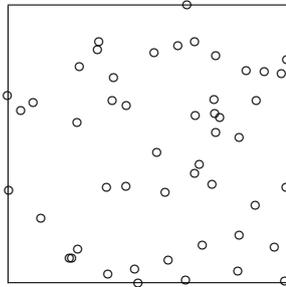
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<sup>1</sup>Cressie [1] defined random quadrats this way, however Ripley [7] used rectangular quadrats that potentially overlapped.

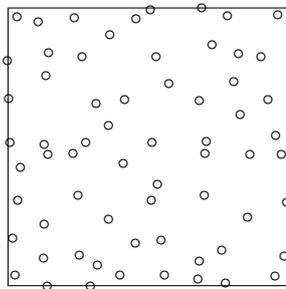
Table 1: Indices for Quadrat Count Data. Adapted from Cressie [1], this table shows the statistics computed for the quadrats in figure 3

Index	Estimator <sup>a</sup>	Realization	Reference
$I$	$\frac{S^2}{\bar{X}}$	2.128	Fisher [3]
$ICS$	$\frac{S^2}{\bar{X}} - 1$	1.128	David and Moore [4]
$ICF$	$\frac{\bar{X}^2}{S^2 - \bar{X}}$	1.383	Douglas [5]
$\bar{X}^*$	$\bar{X} - \frac{S^2}{\bar{X}} - 1$	2.688	Lloyd [6]
$IP$	$\frac{\bar{X}^*}{\bar{X}}$	1.723	Lloyd [6]
$I_\delta$	$\frac{n \sum_{i=1}^n X_i(X_i - 1)}{n\bar{X}(n\bar{X} - 1)}$	1.720	Morisita [7]

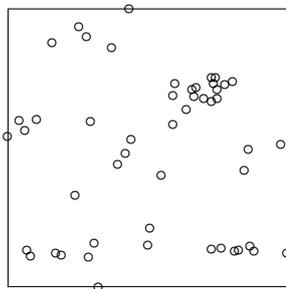
<sup>a</sup> $X_i$  the number of sites in the  $i$ th quadrat,  $\bar{X}$  is the mean of the quadrat counts, and  $S^2$  is the sample variance.



(a)



(b)



(c)

Figure 1: Realization of two-dimensional Poisson processes of 50 points on the unit square exhibiting (a) complete spatial randomness, (b) regularity, and (c) clustering.

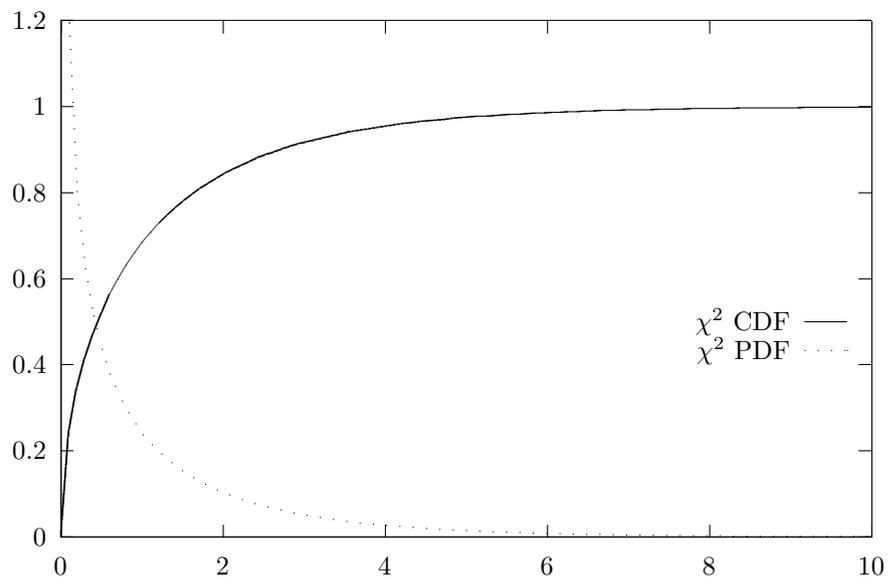


Figure 2: Probability Density Function and Cumulative Distribution Function for a  $\chi^2$  Random Variable.

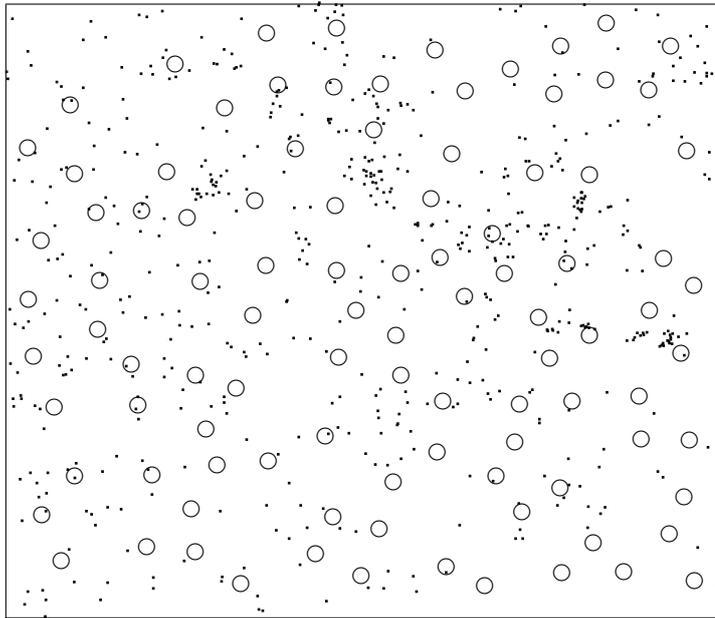


Figure 3: Randomly placed quadrats ( $n = 100$ ) with 584 sample points. This figure was produced using the output of `s.qcount` and `g.gnuplot`.

## References

- [1] Noel A. C. Cressie. *Statistics for Spatial Data*. Wiley Series in Probability and Mathematical Statistics. John Wiley & Sons, New York, NY, 1st edition, 1991.
- [2] F. N. David and P. G. Moore. Notes on contagious distributions in plant populations. *Annals of Botany*, 18:47–53, 1954.
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- [4] R. A. Fisher, H. G. Thornton, and W. A. Mackenzie. The accuracy of the plating method of estimating the density of bacterial populations. *Annals of Applied Biology*, 9:325–359, 1922.
- [5] M. Lloyd. Mean crowding. *Journal of Animal Ecology*, 36:1–30, 1967.
- [6] M. Morista. Measuring the dispersion and analysis of distribution patterns. *Memoires of the Faculty of Science, Kyushu University, Series E. Biology*, 2:215–235, 1959.
- [7] Brian D. Ripley. *Spatial Statistics*. John Wiley & Sons, New York, NY, 1981.